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Time-symmetric relativistic and quantum theories. 2. Time-symmetric relativistic quantum mechanics

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Abstract

Zisman-Stueckelberg-Feynman (ZSF) interpretation, by describing antiparticles as negative-energy particles evolving backward in ordinary time, makes relativistic quantum mechanics (RQM) covariant, but relativistic theory was not agreed with it. Consistent agreement with ZSF interpretation led to the time-symmetric relativity theory (TSRT) (Article 1). In TSRT, relativity principle is extended to rest frames of negative-energy particles going into lower light cone, and symmetry group is extended to the general Lorentz group O(1,3) with 4-inversion. In this article, a time-symmetric RQM (TS RQM) based on TSRT is formulated. The states of negative-energy particles in their rest frames are transformed into usual frames with reflecting time axis. In Hamilton-Jacobi formalism, translational symmetry leads to a general relativistic wave equation for components of wave function of any free particle with a quadratic Hamiltonian. Using in it relativistic relationship between energy and momentum gives Klein-Gordon equation, which yields a general average of 4-momentum and general probability current, time components of which describe currents in both directions of time axis and are negative at negative energy. Special wave equations with first time derivative (Duffin-Kemmer-Petiou for spins 0 and 1, and Dirac for spin 1/2) account for mixing of wave function components due to spin and interactions. In TSRT, 4-vector currents change sign upon 4-inversion, which improves theories of bosons and fermions, eliminating former paradoxes and errors. Time-symmetric transition amplitudes in propagator approach lead to causal propagators and usual diagram technique. Total energy of system of particles in loop diagrams creates external gravitational field, which slows down proper times relative to world time until they freeze at Planck length, gravitational radius of system. This leads to gravitational regularization of loop diagrams, making them finite, and fields of Standard Model (except for scalar field) and quantum gravity renormalizable. Quantization of fields in TSRT is described in Article 3.

Key words: negative energy particles, antiparticles, time inversion, 4-inversion, general Lorentz group, time-symmetric relativity theory, Klein-Gordon equation, Duffin-Kemmer-Petiou equation, Dirac equation

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1. Introduction

The covariant formalism of relativistic quantum mechanics (RQM) includes negative-energy particles. In 1930, Dirac was the first to connect them with antiparticles, put forward hole hypothesis with filled vacuum of negative energy [1]. However, a more correct interpretation was found later by Zisman (1940) [2] and Stueckelberg (1941) [3], who showed that at considering trajectories "as a whole," antiparticles can be described as the same particles, but with negative energy and therefore "going backward" in ordinary time t. Then, in 1949, Feynman [4] created on this basis the propagator approach with causal propagators and developed the covariant diagram technique.

The Zisman-Stueckelberg-Feynman (ZSF) interpretation [2-4] follows directly from relativistic kinematics and is widely used, manifesting itself in particle physics in the form of crossing symmetry. But it was applied inconsistently to RQM and was simply incompatible with quantum field theory (QFT), where the vacuum is the state of lowest energy. ZSF interpretation was thought to lead to negative probability, and there were also problems in fermion theory [5,6]. As will be shown in the article, these problems were consequences of inconsistent application of ZSF interpretation.

The main disadvantage of the former formulation of RQM was that the relativistic theory was still limited to the orthochronous Lorentz group without reflection of the axis t. Consistency with ZSF interpretation requires rest frames of negative-energy particles, where their proper times are measured. Their bases, consisting of antimatter, in terms of negative energies go backwards along the axis t. In particular, in the antihydrogen atom, the antiproton is a basis of the coordinate system, which, when described as a negative-energy proton, is a basis of the reference frame going backwards along the axis t. Time axis of such reference frames is inverse to t, and transitions to the usual reference frame involve a reflection of time axis with a transition to the opposite light cone.

This requires an extension of the principle of relativity and the group of transformations to reference systems going into the lower light cone. For inertial reference frames (IRFs), this group is the general Lorentz group $O(1,3)$, which includes 4-inversion and encompasses both timelike light cones. Including translations leads to the general Poincaré group (see [7]). The basic principles of the time-symmetric relativity theory (TSRT) based on these groups were presented in Article 1 [8]. This article describes a time-symmetric RQM (TS RQM) based on TSRT.

RQM introduces a new concept of wave equations and probability current. For relativistic kinematics, where the Hamiltonian appears quadratically, the Hamilton-Jacobi

formalism has proven most suitable. Lagrangian and Hamiltonian formalisms previously led to complications and unsolvable problems, attempts to solve of which led to a number of errors and contradictions. In RQM, translational symmetry leads to a *general wave equatio*n for the components of wave function of free relativistic particle $-\hbar^2\partial_t^2\psi = H_0^2\psi$, which, at inserting the relationship between energy and momentum, yields Klein-Gordon (KG) equation. $(\partial^\mu\partial_\mu + \kappa^2)\psi = 0$, where $\kappa = mc/\hbar$. It yields the general 4-momentum average and the general probability current proportional to it, time components of which are currents in both directions of time axis and, in particular, are negative for negative energies. This corrects the error of former RQM, where the sign of probability current was attributed to the probability.

Spin and interactions lead to mixing of the wave function components that express *special wave equations* of the form $(i\Gamma_s^\mu D_\mu - \kappa)\psi = 0$ with the first time derivative in the extended derivative D_μ. They follow from the general wave equation at linearization of the relativistic Hamiltonian. For spins $s = 0,1$ this is the Duffin-Kemmer-Petiou (DKP) equation with $\Gamma_{0,1}^\mu = \beta_{0,1}^\mu$ [9-11] (see [6,12]), and for spin $s = 1/2$, this is the Dirac equation with $\Gamma_{1/2}^\mu = \gamma^\mu$. The Feshbach-Villars formalism for spin 0 [13] was only the first step towards the DKP formalism and led to a number of problems.

In Dirac's theory, the four-vectors of the probability current and electromagnetic current for a positive charge were inverse to the four-momentum of a negative-energy particle, which was considered as an achievement of the theory. In reality, this was a failure of the theory, since in relativistic theory, the signs of these currents must coincide with the sign of 4-momentum. In TS RQM the signs of these currents coincide with the sign of 4-momentum and this makes the theory of fermions consistent, also excluding the former paradoxes.

Time-symmetric transition amplitudes of TS RQM in the propagator approach lead to causal propagators and to the usual diagram technique.

In loop diagrams, the main problem of relativistic quantum theory was ultraviolet divergences. This problem arises already in RQM and therefore should have been solved within its framework, not arising further in QFT. In TS RQM, it is solved by taking into account the external gravitational field required by general relativity (GR), created by a system of particles in a loop diagram [14]. This field leads to a slowing down of the proper times of particles relative to the world time of an asymptotically flat spacetime, and near the Planck length, the gravitational radius of the particle system, all processes freeze. Such gravitational self-regularization of loop diagrams at the Planck length makes them finite. Since the effective coupling constants of the gauge fields of the Standard Model (SM) and quantum gravity remain small, these theories also turn out to be renormalizable.

In the article, the velocity of light c and Planck's constant $\hbar$ are shown, since for bosons, at $c = 1$ the time component of the probability *current* density $j_\pm^0 = (\pm c)\rho$ was considered as a probability density $\pm\rho$, and it was believed that RQM leads to negative probability.

In Section 2, basics of TSRT, wave functions, general wave equation, and general probability current are considered. In Sections 3-4, special wave equations for bosons and fermions and their solutions, as well as interaction currents, are considered. In Section 5, causal propagators and diagram technique are described. In Section 6, gravitational self-regularization of loop diagrams is described. In the Article 3 [15], a time-symmetric QFT based on TSRT will be presented. A preliminary formulation of the main ideas of TS RQM were described in [16], and a more detailed presentation is given in the book [17].

2. TSRT, general probability current, general and special wave equations

2.1. Reference frames in TSRT, orientable hypersurfaces and probability current in two directions of time axis

In the former RQM, negative-energy particles were described only in usual rest frames K_+ with ordinary time axis t_+ (hereinafter t). In TSRT, negative-energy particles are also described in rest frames K_{0-}, where their proper times τ_- are measured. They move backward along the axis t, and their time axis t_{0-} is inverse to t. For easily transition to K_+, a rest frame K_- is also introduced, which, like K_{0-}, moves backward along the t axis, but is at rest in space relative to K_+ with proper time axis $t_- = -t$. The coordinates of the same event in K_+ and K_- are related by 4-inversion: $x^\mu_- = -x^\mu_+$.

Thus, in TSRT, the negative-energy states are first specified in their rest frames K_{0-}, then transformed by proper Lorentz transformations into K_-, and then transformed by 4-inversion into K_+, where all observations are made. The result of these proper and improper transformations from the general Lorentz group $O(1,3)$ is a description of particles of both energy signs in coordinates K_+.

The operators of these transformations commute [7] and they can be rearranged, i.e. one can use 4-inversion to go from K_{0-} to a system K'_+ moving relative to K_+, and then from this system, using proper transformations, go to K_+.

In relativistic mechanics, according the principle of least action, the integral along a trajectory with respect to proper time must be maximal. At negative energy, the integral still between events $\tau_1 > \tau_0$ on the trajectory, but now $t(\tau_1) < t(\tau_0)$ and the integration with respect to t is from the present to the past. Rearranging the limits changes the sign of the time integral, but the minus sign from $dt_- = -dt$ compensates for this sign, ultimately leading to the usual integration from the present to the future.

Varying the action by changing the upper limit in the integral along the true trajectories $\delta S_\pm = -p_{\mu\pm}\delta x^\mu_\pm$ gives for 4-momentum and the Hamiltonian:

$$p_{\mu\pm} = -\frac{\partial S_\pm}{\partial x^\mu_\pm}, \qquad H_\pm = cp_{0\pm}(p_i) = -\frac{\partial S_\pm}{\partial t_\pm}. \tag{1}$$

From $u_{\mu\pm}u^\mu_\pm = 1$ it follows $p_{\mu\pm}p^\mu_\pm = m^2c^2$, and substituting $p_{\mu\pm}$ from (1) leads to the relativistic Hamilton-Jacobi equation. The Hamiltonian enters this equation quadratically, and the expression for it in terms of the 3-momentum $\vec{p}$ has two signs. In quantum theory, this leads to the general wave equation with a second time derivative.

In RQM, probabilities are positive, and currents through spacelike hypersurfaces Σ_μ, including the probability current and interaction currents, change sign when the energy changes sign. This follows from the fact that particles of these flows intersect Σ_μ from different directions, and the projection of their 4-velocities onto the timelike normal $\mathbf{n}_+$ has different signs. The currents also change sign when the t axis is reflected, since for the same current, the normal onto which the current vector is projected changes.

In relativistic theory, hypersurfaces Σ_μ have two sides facing the future and the past, respectively. The normal $\mathbf{n}_+$ to one side points toward the upper light cone, and the normal $\mathbf{n}_-$ to the other side points toward the lower light cone. In usual relativistic theory, only hypersurfaces $\Sigma_{\mu+}$ with a normal $\mathbf{n}_+$ were introduced, and therefore their orientation was trivial. The presence of a second type of reference frame, pointing toward the lower light cone with the axis t_- inverse to t, means the presence of a hypersurface $\Sigma_{\mu-}$ with a normal $\mathbf{n}_-$, and therefore, in TSRT, hypersurfaces Σ_μ have a nontrivial orientation.

Let us consider a hypersurface Σ with an oriented element $d\Sigma_\mu = n_\mu d^3\Sigma,$ where n_μ are the components of the timelike unit normal with $n^\mu n_\mu = 1$, and $d^3\Sigma > 0$ is an element of the unoriented (usual) 3-volume. The invariant total probability current through Σ is the integral over Σ from the probability current density vector j^μ:

$$P[\Sigma] = \int\limits_\Sigma d\Sigma_\mu j^\mu = \int\limits_\Sigma d^3\Sigma (n_\mu j^\mu). \tag{2}$$

From $\partial_\mu j^\mu = 0$ independence of $P[\Sigma]$ from the choice Σ (usual boundary conditions) follows. In the particle's rest frame (2), the integral over the hypersurface $t = const$ in K_+ has the form:

$$P[\Sigma] = \int\limits_\Sigma d\Sigma_0 j^0 = \int\limits_{V_{(0)}} dV_{(0)} (n_0 j^0), \tag{3}$$

where $V_{(0)}$ is an invariant and sufficiently large proper normalization volume.

In the former RQM, negativity j^0 was mistakenly considered as the negative probability density. Since negative probability is unacceptable, 4-vector j^μ was multiplied by the absolute value of the charge and interpreted as the interaction current density. The answer to the question about the probability current was referred to QFT, thereby acknowledging the inconsistency of RQM boson theory.

In TS RQM j^μ is considered to be exactly what it is: a 4-vector of probability current, the sign of which is determined by 4-momentum. Specifically, the sign j^0 indicates the direction of the current along time axis. The proper probability current in the rest frame Σ is expressed through the proper probability density $\rho_\Sigma(x) \geq 0$:

$$c\rho_\Sigma(x) = n_\mu j^\mu(x). \tag{4}$$

In 4-inversion, the signs of both j^μ and the oriented element $d\Sigma_\mu$ change:

$$j^\mu \mapsto -j^\mu, \qquad d\Sigma_\mu \mapsto -d\Sigma_\mu, \tag{5}$$

and therefore the integral $P[\Sigma]$ in (2) and $\rho_\Sigma = n_\mu j^\mu$ remain positive definite.

In relativistic theory, the components of 4-velocity $u^\mu = dx^\mu / ds$ are the components of the tangent vector to the trajectory, and they are used to determine the components of a number of other tensor quantities. In quantum theory, the probability densities of these quantities are defined, expressed through bilinear forms of the wave function. In particular, the energy-momentum tensor of a particle current, which has the form $T^{\mu\nu} = \varepsilon_{(0)} u^\mu u^\nu$ in the

absence of pressure, includes the invariant proper energy density $\varepsilon_{(0)} = dE_{(0)} / dV_{(0)}$ and two 4-velocities, making $T^{\mu\nu}$ a second-rank tensor.

The probability current density vector j^{μ}, like the electromagnetic current and $T^{\mu\nu}$, is defined as the product of the invariant proper probability density $\rho_{(0)} = dw / dV_{(0)}$ and cu^{μ}:

$$j^{\mu} = \rho_{(0)} c\, u^{\mu} = \frac{1}{m} \rho_{(0)} mcu^{\mu}. \tag{6}$$

In TSRT, the important fact is that $\rho_{(0)}$ is the probability density measured in the particle's rest frame, while 4-velocity u^{μ} is measured in K_{+}, where the particle is moving. Since the sign of 4-velocity determines the sign of 4-momentum, the sign of j^0 coincides with the sign of the energy. Therefore, in TSRT, probabilities are positive and invariant, but their current j^0 has two signs depending on the direction of intersection of Σ_{0+}, the hypersurface of simultaneity $t = const$.

2.2. Invariant and laboratory wave functions, physical quantities

In nonrelativistic quantum mechanics, the complex wave function in exponential form $\psi(x,t) = \sqrt{\rho(x,t)} \exp[iS_{nr}(x,t)/\hbar]$ includes a probability density $\rho = |\psi|^2$ and a phase function S_{nr}. The gradient of the latter gives the drift momentum $\vec{\nabla} S_{nr} = m\vec{v} = \vec{p}$, and in the semiclassical limit S_{nr} is related by the particle action function.

The relativistic wave function in the rest frame coincides with the nonrelativistic one up to phase and spinor factors. We therefore begin the introduction of wave functions into RQM with the particle's rest frame $K_{(0)}$, where the wave function $\psi_{(0)}$ is associated with the proper probability density $\rho_{(0)}$ in $V_{(0)}$ and has the form:

$$\psi_{(0)}(x_{(0)}) = \sqrt{\rho_{(0)}(x_{(0)})}\, u_{(0)} e^{iS_{(0)}(x_{(0)})/\hbar}. \tag{7}$$

It includes only invariant quantities: $\rho_{(0)}$, (iso)spinor $u_{(0)}$ in the rest frame and the phase $S_{(0)} / \hbar$, which includes the addition due to the rest energy $mc^2 t / \hbar$. Therefore, $\psi_{(0)}$ is invariant and will hereafter be called *the invariant wave function* of the particle.

The total probability in volume $V_{(0)}$ and bilinear form $\bar{\psi}_{(0)}\psi_{(0)}$ are equal to:

$$w(V_{(0)}) = \int\limits_{V_0} dV_{(0)} \frac{dw}{dV_{(0)}} = \int\limits_{V_0} dV_{(0)}\, \bar{\psi}_{(0)}\psi_{(0)}, \tag{8}$$

$$\bar{\psi}_{(0)}\psi_{(0)} = \rho_{(0)}\bar{u}_{(0)}u_{(0)}, \tag{9}$$

Here $w(x,t)$ is the local probability, the total probability in $V_{(0)}$ is normalized $w(V_{(0)}) = 1$ under the condition $\bar{u}_{(0)}u_{(0)} = 1$, and $\bar{\psi}$ denotes $\psi^{+}\Gamma_s^0$, where $\Gamma_s^0 = \beta_s^0$ for spins 0 and 1 in the DKP formulation, and $\Gamma_{1/2}^0 = \gamma^0$ for spin ½ (see Sections 3-4).

The wave function ψ in the laboratory frames $K_{\pm}$, in general case moving relative to $K_{(0)}$, differs from $\psi_{(0)}$ due to tensor (spinor) properties and the volume transformation

$V_{(0)} \to V(V_{(0)})$. For this reason, we will call it *the laboratory wave function*. At transforming to, K_+ the relationship (8) takes the form:

$$w = \int\limits_{V_0} dV_{(0)}\, \bar{\psi}_{(0)}\psi_{(0)} = \frac{V_{(0)}}{V} \int\limits_{V(V_0)} dV\, \bar{\psi}\psi \frac{\bar{\psi}_{(0)}\psi_{(0)}}{\bar{\psi}\psi}. \tag{10}$$

Here, integration over the laboratory volume V occurs only in that part of the space that is related with its proper volume V_0 from $K_{(0)}$ with the relativistic relation. At transforming from $K_{(0)-}$ to K_+ and the condition, $\bar{\psi}_0\psi_0 / \bar{\psi}\psi = const$, the relation (10) takes the form:

$$w = \frac{V_{(0)}}{V} \frac{\bar{\psi}_{(0)}\psi_{(0)}}{\bar{\psi}_-\psi_-} \int\limits_{V(V_0)} dV\, \bar{\psi}_-\psi_- = \frac{\bar{u}_{(0)}u_{(0)}}{\bar{v}v} \int\limits_{V(V_0)} dV\, \bar{\psi}_-\psi_-, \tag{11}$$

where v is the spinor K_+ for a negative energy state.

Former normalization $V = 1$ in all reference frames masked the fact that bilinear forms yield probability densities and distorted the transformation properties of these forms. Since only V_0 is invariant in relativistic theory, only normalization $V_0 = 1$ is natural, which yields $V = mc^2 / E_{p\pm}$. Therefore, in relativistic theory, normalization in a finite volume is necessary to account for the transformation properties of ψ.

In quantum theory, physical quantities correspond to operators F, and their measurable quantities are their eigenvalues f in states ψ. These eigenvalues are determined from the equation $F\psi = f\psi$, where the values f, due to the homogeneity of the equation, do not depend on the normalization constants ψ. It is more convenient to find these eigenvalues from the matrix element with the operator of the physical quantity:

$$\bar{\psi}F\psi = f\bar{\psi}\psi \tag{12}$$

As can be seen from (12), $\bar{\psi}F\psi$ gives not the values f themselves, but multiplied by the normalization factor $\bar{\psi}\psi$ at its nonzero points $\bar{\psi}\psi \neq 0$. Therefore, f are equal to the matrix element divided by this (non-zero) normalization factor:

$$f = \frac{\bar{\psi}F\psi}{\bar{\psi}\psi}. \tag{13}$$

Further, taking this fact, obvious at first look, into account in TS RQM will allow us to correct a number of errors in the theory of fermions of the former RQM, making RQM consistent.

Let us consider the consequences of the fact that in TS RQM physical quantities are determined by two types of wave functions: $\rho_{(0)}$ is given by the invariant wave function $\psi_{(0)}$ in the particle's rest frame, and u^μ is determined by the laboratory wave function ψ in K_+. When ψ is represented in exponential form as in (7), the eigenvalues of the drift (real) part of 4-momentum operator mcu^μ are expressed through the complex 4-momentum operator $i\hbar\partial^\mu$, determined from translational symmetry:

$$\frac{i\hbar}{2}[\bar{\psi}\,\partial^\mu\psi - (\partial^\mu\bar{\psi})\psi] = mcu^\mu\bar{\psi}\psi, \tag{14}$$

$$mcu^\mu = \frac{i\hbar}{2}\frac{\bar{\psi}\,\partial^\mu\psi - (\partial^\mu\bar{\psi})\psi}{\bar{\psi}\psi}. \tag{15}$$

This gives for j^μ from (6), using $\rho_0 = \bar{\psi}_0\psi_0$, the expression:

$$j^\mu = \frac{1}{m}\rho_0 mcu^\mu = \frac{i\hbar}{2m}\frac{\bar{\psi}_0\psi_0}{\bar{\psi}\psi}[\bar{\psi}\partial^\mu\psi - (\partial^\mu\bar{\psi})\psi]. \tag{16}$$

When $\bar{\psi}_0\psi_0 / \bar{\psi}\psi$ does not depend on the coordinates, (16) leads to the continuity equation:

$$\partial_\mu j^\mu = 0. \tag{17}$$

Thus, in TS RQM the probability current density j^μ is expressed through $\bar{\psi}_0\psi_0$ and mcu^μ, the drift part of 4-momentum (15), and therefore the final expression j^μ includes the normalization factor $\bar{\psi}_0\psi_0 / \bar{\psi}\psi$. For free particles of positive energy, it is equal to the ratio of probability densities and reduces to the volume factor: $\bar{\psi}_0\psi_0 / \bar{\psi}\psi = V / V_0 = mc^2 / E_p$. The importance of accurately determining the current density is not limited to refining the contribution of the volume factor and will manifest itself in fermion theory in *the difference in the sign,* which radically changes the situation with the probabilistic interpretation.

In the next section, it will be shown that the general wave equation (22) for separate components of the wave function of free particles of all types leads to j^μ in (16) and to the continuity equation (17), which expresses the conservation of probability. For this reason, j^μ becomes *the general probability current* for any free particles.

In TSRT, the time component j^0 is a current along time axis, and its sign coincides with the sign of the energy [8]. In TSRT, this resolves former problems with signs of the probability density for bosons and the probability current density for fermions. In Section 4.3, it will be shown that taking into account the properties of spinor transformations leads to the fact that at negative energies, the signs of the eigenvalues of the probability current and interaction currents are the same as for bosons and are consistent with the requirements of TSRT.

Expressions for the special interaction currents follow from the special wave equations. Interaction with the gauge field is introduced by extending the derivatives or 4-momenta, ensuring gauge invariance of the equations. For the electromagnetic field potential A^μ has the form: $D^\mu_\pm = \partial^\mu_\pm + ieA^\mu / \hbar c$. Interaction currents differ from the probability current (16) by multiplication by charges and transformation properties under 4-inversion, and can also contain field potentials.

The components of the interaction currents j^0_e change sign with charge. In RQM, where there are no antiparticles with charges opposite to those of the particles, the situation is simpler, and instead of antiparticles, there are particles with negative energy having the same charge as the particles, but evolving backward in ordinary time.

In cases where differences between the former and new expressions for the probability current for free particles affect measurable quantities, TS RQM leads to new observable effects (see [17]).

2.3. General wave equation for relativistic particles

The wave equations in K_+ and K_- for particles moving forward in times of these reference frames have the same form, as required by the principle of relativity, expressed in invariance with respect to the group $O(1,3)$. Therefore, we write these equations and their solutions in K_+ and K_-, denoting them by the index $\pm$ if necessary.

Components of the wave function of a free particle satisfies the Schrödinger equation, which follows from translational symmetry:

$$i\hbar\partial_t\psi = H_0\psi. \tag{18}$$

At the exponential representation like (7), the Hamilton-Jacobi-Madelung equation $H_0 = -\partial S / \partial t$ for the phase of the wave function, an analog of the action function, follows. Moreover, unlike the classical Hamiltonian, H_0 contains both a kinetic term with drift momentum and a purely quantum term expressing the spreading rate of the wave packet. Thus, in quantum theory, translational symmetry leads directly to an analog of the Hamilton-Jacobi formalism.

In relativistic kinematics, the square of 3-momentum $\vec{p}^2$ is proportional not to H_0, but to its square:

$$H_0^2 = c^2(\vec{p}^2 + m^2c^2). \tag{19}$$

As the result, in RQM, it is not easy to define the Hamiltonian, since at inserting $\vec{p}^2 \sim \nabla^2$ a direct definition H_0 as a root of the expression (19) leads to a non-local theory with higher derivatives.

How to consistently proceed from the relativistic Hamiltonian (19) to the Eq. (18), but with first derivatives with respect to spatial coordinates also. This requires introducing a matrix Hamiltonian, which therefore acts on a multicomponent wave function, and this is realized in the form of special wave equations DKP and Dirac equations. The matrix Hamiltonian is different for different spins and mixes the components of the wave functions. These special equations will be discussed in the next section.

In this section, we consider the general wave equation for relativistic particles, which is simpler but also more universal, as it is valid for individual components of the wave function of any free particle. It follows from the Schrödinger form of the equation (18) by repeated acting by the operator $i\hbar\partial_t$:

$$-\hbar^2\partial_t^2\psi = H_0^2\psi. \tag{20}$$

Here, the free Hamiltonian appears in quadratic form, which reveals the specific nature of relativistic kinematics. It is analogous to the relativistic form of the Hamilton-Jacobi equation $H_0^2 = (\partial S / \partial t)^2$, where substituting the momentum along the true trajectories $\vec{p} = \partial S / \partial \vec{x}$ and (19) yields $(\partial S / c\partial t)^2 - (\partial S / \partial \vec{x})^2 = m^2c^2$ or $(\partial S / \partial x^\mu)(\partial S / \partial x_\mu) = m^2c^2$.

For stationary states Eq. (20) gives an equation with the square of the energy eigenvalues $E_{p\pm} = \pm E_p$, $E_p = c\sqrt{\vec{p}_\pm^2 + m^2c^2}$:

$$H_{(0)}^2\psi_i = E_{p\pm}^2\psi_i. \tag{21}$$

This, together with (18), implies that the eigenvalues of the Hamiltonian are $E_{p\pm} = \pm E_p$, and there are states of two energy signs. Thus, the presence of two signs $E_{p\pm}$ leads to a doubling of the state space and the time-symmetric formalism of TS RQM.

Substituting Eq. (19) and $\vec{p} = i\hbar\nabla$ into (20) leads to the KG equation:

$$(\partial_\pm^\mu\partial_{\mu\pm} + \kappa^2)\psi_{i\pm} = 0. \tag{22}$$

The KG equation expresses the relativistic relationship between energy and momentum and therefore, being a differential form of the general wave equation (19), is valid for the components of the wave function of any free relativistic particle.

The universality of the KG equation (22), known from the formation of RQM, acquires fundamental importance in TS RQM, since it leads to a general average of 4-momentum p^μ and to a general probability current j^μ, as well as the conservation equations for these two most important vectors of RQM.

The KG equation (22) includes a second time derivative $\partial_t^2\psi$, and therefore it was believed that initial values should also be specified for $\partial_t\psi$. Arbitrariness of $\partial_t\psi$ in RQM would naturally radically revise the theory, particularly for the probability current. However, this statement was erroneous, since such arbitrariness does not exist in quantum theory. In RQM, the Schrödinger equation (18) remains valid, and specifying the initial wave function automatically determines, through the Hamiltonian, its time derivative.

Therefore, as noted above, the question was how to construct a local Hamiltonian. For a single-component wave function, this proved impossible, and the transition to an equation (18) led to multicomponent wave functions, different for different spins and satisfying different special wave equations.

But this turned out to be true only partially, since when the eigenvalues of the square of the energy H_0^2 for stationary states are known from (21), then $i\partial_t\psi_0 = E_{p\pm}\psi_0$ is valid for them. This means a larger limitation on the solutions of the first-order equation (18), which distinguishes RQM from a theory with a pure second-order equation (22) without any restrictions on $\partial_t\psi$. For this reason, the time dependence of free wave functions is also universal, does not depend on spins, and follows directly from the general equation (22).

2.4. Special wave equations for relativistic particles

The general wave equation (20) contains $\partial_t^2\psi$, which is why initial values for $\partial_t\psi$ must also be formally specified. Furthermore, Eq. (20) contains not H_0, but its square $H_{0\pm}^2$. Therefore, it was necessary to transform (20) into Schrödinger form, to a special wave equation with $\partial_t\psi$ and $H_{0\pm}$. As noted above, such *special wave equations* have the general form:

$$(i\,\Gamma_s^\mu D_\mu - \kappa)\psi = 0, \tag{23}$$

where s is the spin. For bosons of spin 0 and 1 we have $\Gamma_{0,1}^\mu = \beta_{0,1}^\mu$, where β_0^μ and β_1^μ are the DKP matrices (see Section 3), and for fermions $\Gamma_{1/2}^\mu = \gamma^\mu$, the Dirac matrices.

If in the case of the general wave equation the Hamiltonian-Jacobi formalism appears as suitable to the specifics of relativistic theory, easily operating with the square of the Hamiltonian, the usual Lagrangian and Hamiltonian formalisms can be used in the case of special wave equations in Schrödinger form operating with the Hamiltonian itself. As it is known, the special wave equations follow from the Lagrangian:

$$\mathcal{L} = c\bar{\psi}(i\hbar\Gamma_s^\mu D_\mu - mc)\psi. \tag{24}$$

which gives the corresponding Hamiltonian.

These special equations impose constraints on the solutions of the general equation for the following reasons. First, they explicitly remove arbitrariness in the choice of initial values for $\partial_t\psi$, in particular, confirming the universality of the time dependence of stationary states. Second, they take into account mixing of the wave function components due to spin and other degrees of freedom, as well as interactions. Thus, special wave equations are more concrete, expressing the specificity of each spin and charge state, and therefore underlie the practical applications of the theory.

The special wave equations for spin 0 and 1 bosons are the DKP equations, which will be discussed in Section 3.1. In this representation, the wave function for a spin 0 particle is a 5-

component spinor, where one component is the scalar density and the other four are its derivatives. The wave function for a spin 1 particle is a 10-component spinor, where four components form the vector and the other six components are equal to the components of the stress tensor for this vector.

A special wave equation for spin ½ is the Dirac equation, which first demonstrated that spin follows from relativistic theory. However, this equation yielded probability currents and interaction currents that, contrary to relativistic requirements, did not change sign at changing the sign of energy. As noted in Section 2.3 and will be shown in more detail in Section 4.3, this shortcoming of Dirac's theory is corrected by more carefully defining the eigenvalues of the operators, which then leads for the first time to a consistent theory of fermions.

Thus, for each type of relativistic particle, there are two wave equations: the separate components of the wave functions satisfy a general wave equation (20) with a second time derivative, leading to a general average energy and probability current for free particles (16), and the special wave equations with a first derivative (23) express mixing of the components associated with the spin and interactions of the particles.

3. Time-symmetric theory of bosons

3.1. The DKP special wave equation for particles of spin 0 and 1

The well known and most successful of the special wave equations is the Dirac equation for spin ½ fermions. The bosonic analogue of this equation is the DKP equation [9-11] ([6,12]) with $\Gamma^{\mu}_{0,1} = \beta^{\mu}_{0,1}$ in (24), a special wave equation for finite mass bosons with spins 0 and 1:

$$(i\beta^{\mu}_{s} D_{\mu} - \kappa)\psi = 0. \tag{25}$$

In the DKP formalism, the Dirac linearization method is applied to the case of bosons. Relativistic invariance and the equivalence of the squared equation to the KG/Proca equations determine the properties of the matrices β^{μ}_{s}. They form a basis for the Duffin–Kemmer algebra with cubic anticommutation relations:

$$\beta^{\mu}_{s}\beta^{\nu}_{s}\beta^{\lambda}_{s} + \beta^{\lambda}_{s}\beta^{\nu}_{s}\beta^{\mu}_{s} = g^{\mu\nu}\beta^{\lambda}_{s} + g^{\lambda\nu}\beta^{\mu}_{s}. \tag{26}$$

In the descriptions of RQM, the FW formalism was usually presented [13] (see [5,6]), where $\partial_t\psi$ is considered to be the second component of the new wave function and the general wave equation transformed into two equations with the first order time derivative. This approach is only the first step towards the DCT formalism and for this reason led to a number of problems: a) the wave equations contain $\vec{\nabla}^2$ and are clearly non-covariant; b) the Hamiltonian is non-Hermitian and therefore the evolution operator is not unitary; c) a nonlinear Foldy-Wouthuysen representation is required and physical problems are solved approximately; d) the interactions are introduced non-covariantly, since they included A^0 and $\vec{A}^2$.

All these problems are absent in the DKP formalism, in which all four derivatives of the wave function $\partial_{\mu}\psi_i$ become components of a new wave function, 5-component for spin 0 and 10-component for spin 1. The wave function ψ_i there is a column $(\psi_1,...,\psi_5)^T$ for spin 0 and $(\psi_1,...,\psi_{10})^T$ for spin 1, and the general wave equation (23), after selecting the number of components and matrices β^{μ} (5×5 and 10×10), gives a system of covariant equations (25) with first derivatives only.

The advantages of the DKP approach are: a) a unified system for spins 0 and 1 based on the same matrix algebra β^{μ}_{s}; b) explicit covariance due to coordinate symmetry; c) the Hamiltonian is Hermitian; d) covariant introduction of interactions; d) built-in couplings for

components that allow reduction to the KG/Proca equations; e) there is no gap between the formalisms for fermions and bosons; g) simplifies calculations in the diagram technique due to the presence of matrix algebra techniques as in fermion theory.

The only fundamental drawback of the DKP formalism was that it seemed to lead to negative probabilities. For this reason, it was supposed that boson theory could only be physically consistent within QFT. But this drawback disappears in TS RQM, where probabilities are positive, and the signs of the average energy and probability current indicate the direction of evolution along the t axis. Let us consider the basics of the DKP formalism and the modifications introduced by TS RQM.

For spin 0 the matrices β_s^μ have the form:

$$\beta_0^\mu = \begin{pmatrix} 0 & \delta_{\mu 0} & -\delta_{\mu 1} & -\delta_{\mu 2} & -\delta_{\mu 3} \\ \delta_{\mu 0} & 0 & 0 & 0 & 0 \\ \delta_{\mu 1} & 0 & 0 & 0 & 0 \\ \delta_{\mu 2} & 0 & 0 & 0 & 0 \\ \delta_{\mu 3} & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{27}$$

The wave function in this representation takes the form $\psi = (\psi^0, \psi^\mu)^T$

In the case of spin 1, the matrices β_s^μ can be represented in block form:

$$\beta_1^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^T & 0_3 & I_3 & 0_3 \\ \bar{0}^T & I_3 & 0_3 & 0_3 \\ \bar{0}^T & 0_3 & 0_3 & 0_3 \end{pmatrix}, \qquad \beta_1^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^T & 0_3 & 0_3 & -iS_i \\ -e_i^T & 0_3 & 0_3 & 0_3 \\ \bar{0}^T & -iS_i & 0_3 & 0_3 \end{pmatrix}. \tag{28}$$

Here I_3 is the unit 3×3 matrix, 0_3 is the zero 3×3 matrix, $\bar{0}$ is the zero row 1×3, e_i are unit vectors-rows of the basis $e_1 = (1,0,0)$ and so on, S_i are the 3×3 spin matrices for the spin 1, with $(S_k)_{mn} = -i\epsilon_{kmn}$. The wave function in this representation has the form $\psi = (\psi^0, \vec{\psi}, \vec{\pi}, \vec{E})^T$, where the first 4 components correspond to 4-potential, and the next 6 components correspond to the antisymmetric field tensor $F_{\mu\nu}$.

In the case of spin 0, the matrix β_0^0 relates the first and second components of the wave function. The equation of motion splits, taking the form:

$$(i\partial_0 - \kappa)\psi_1 = \kappa\psi_2 \Rightarrow \psi_2 \sim \dot{\psi}_1. \tag{29}$$

This confirms the connection with the second-order equation: ψ_1 plays the role of the scalar density ϕ, and $\psi_2, \psi_3, \psi_4, \psi_5$ are related (up to factors) with gradients $\partial_\mu \phi$.

The adjoint spinor leading to the correct covariant bilinear forms is defined as:

$$\bar{\psi} = \psi^\dagger \eta_s^0, \qquad \eta_s^0 = 2\beta_s^0 \beta_s^0 - \mathbb{I}. \tag{30}$$

The Lagrangian density with $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu / \hbar c$ has the form:

$$\mathcal{L} = c\bar{\psi}(i\hbar \beta_s^\mu D_\mu - mc)\psi. \tag{31}$$

The proper transformation $x' = \Lambda_{\pm} x$, where $\Lambda_+ \in O^{\uparrow}(1,3)$ and $\Lambda_- \in O^{\downarrow}(1,3)$ leads to a transformation of spinors with the transformation matrix S_s:

$$\psi'(x') = S_s(\Lambda)\psi(x), \qquad S_s^{-1}(\Lambda)\beta_s^{\mu} S_s(\Lambda) = \Lambda_{\nu}^{\mu}\beta_s^{\mu}, \tag{32}$$

$$S_s(\Lambda) = \sqrt{\frac{V}{V'}}\exp\left(-\frac{i}{2\hbar}\omega_{\mu\nu}\Sigma_s^{\mu\nu}\right), \qquad \Sigma_s^{\mu\nu} = \hbar(\beta_s^{\mu}\beta_s^{\nu} - \beta_s^{\nu}\beta_s^{\mu}). \tag{33}$$

Here $\Sigma_s^{\mu\nu}$ is the spin tensor, the generator of the proper Lorentz transformations.
The extension to discrete elements, including 4-inversion, leads to covariance with respect to the general group $O(1,3)$.

The symmetry $\psi \mapsto e^{i\alpha}\psi$ leads to a 4-vector probability current:

$$j_s^{\mu} = \frac{c}{2}\bar{\psi}\,\beta_s^{\mu}\psi, \qquad \partial_{\mu} j_s^{\mu} = 0. \tag{34}$$

The analogue of the Gordon expansion for the probability current is:

$$j_s^{\mu} = c\bar{\psi}\,\beta_s^{\mu}\psi = \frac{1}{2m}[i\hbar\bar{\psi}\,\ddot{\partial}^{\mu}\psi + i\partial_{\nu}(\bar{\psi}\,\Sigma_s^{\mu\nu}\psi)]. \tag{35}$$

The current j_s^{μ}, as in Dirac's theory, is divided into two parts:

a) convective (drift) part:

$$j_{\text{conv}}^{\mu} = \frac{i\hbar}{2m}\bar{\psi}\,\ddot{\partial}^{\mu}\psi, \tag{36}$$

has two signs depending on the sign of 4-momentum, in particular, the sign j_{conv}^0 coincides with the sign of the energy, which is required by TSRT;

b) spin/polarization part (in the form of divergence):

$$j_{\text{s,pol}}^{\mu} = \frac{1}{2m}\partial_{\nu}(\bar{\psi}\,\Sigma_s^{\mu\nu}\psi), \tag{37}$$

does not contribute to $\partial_{\mu} j_s^{\mu}$, since the operator $\partial_{\mu}\partial_{\nu}$ is symmetric, but $\Sigma_s^{\mu\nu}$ antisymmetric.

In the momentum representation for the spinor part of the current we obtain:

$$c\bar{u}(p')\beta_s^{\mu}u(p) = \frac{1}{2m}\bar{u}(p')[(p'^{\mu} + p^{\mu}) + i\Sigma_s^{\mu\nu}(p'_{\nu} - p_{\nu})]u(p). \tag{38}$$

For a particle of spin 0, the probability current coincides with the general probability current (16), since in this case there is no contribution from the spin part and the matrix element (38) is:

$$c\bar{u}(p')\beta_s^{\mu}u(p) = \frac{1}{2m}\bar{u}(p')(p'^{\mu} + p^{\mu})u(p). \tag{39}$$

In the case of spin 1, the probability current (38) for free particles with $p'^{\mu} = p^{\mu}$ takes the form (39), coinciding with the general current of probability (16).

For charged particles, the interaction current A_{μ} is defined as $e\,j_s^{\mu}$ and the sign of this current depends on the combination of signs j_s^{μ} and $e = \pm|e|$.

Let $\Lambda \in O(1,3)$, $\Lambda^T\eta\Lambda = \eta$ and $\det\Lambda = \pm 1$, i.e., the time orientation reflection is permissible. Then the covariance of the DKP equation is given by the condition (32) on $S_s(\Lambda)$ and for $\Lambda = -I$ we have $x'^{\mu} = -x^{\mu}$, $\partial'_{\mu} = -\partial_{\mu}$. If there exists a matrix $S_{(4)}$ such that

$$S_{(4)}^{-1}\beta_s^\mu S_{(4)} = -\beta_s^\mu \tag{40}$$

then the DKP equations are invariant under 4-inversion. The specific implementation $S_{(4)}$ depends on the chosen representation (5- or 10-dimensional).

Discrete transformations (including C, P, T and their combinations) have been studied in the literature for different interactions [6,12]. At 4-inversion $\Lambda = -I$ 4-vectors of the probability current j_s^μ and interaction current $e\,j_s^\mu$ change sign, the continuity equation remains unchanged: $\partial'_\mu j'^\mu_s = \partial_\mu j_s^\mu = 0.$

The DKP spinor contains redundant components that are eliminated by consistent auxiliary conditions arising from the DKP equations, providing a reduction to the KG/Proca equations, and this is considered a key difference from the Dirac equation [6,12]. The equivalence of the observables with the KG/Proca results under minimal coupling (with a correct interpretation of the components and taking into account the couplings) and the mass shell condition is rigorously proven by considering S the-matrix for the scalar sector.

The DKP equations have been solved for a number of important potentials, such as the oscillatory and Coulomb potentials, and their energy levels are generally consistent with the results of the KG/Proca equation, the difference being mainly due to the approximations used.

3.2. Transformation of wave functions and the probability current from the reference frame going back in time to the usual frame

Let's look at the standard relationships for particles of positive energy in K_+, and also, due to invariance with respect to the group of transformations $O(1,3)$, the same relations for particles of negative energy, but now in the inverse reference frame (IRF) K_- moving backward along the t axis. In this case, energies negative in K_+ will become positive in K_- and vice versa. The coordinates K_+ and K_- are related by 4-inversion, and therefore we will write all the relations together, taking into account that they refer to two inverse reference frames (IRF) with reversed time axes.

Wave functions $\psi_\pm$ satisfy the general and special wave equations and are transformed under 4-inversion to describe particles with energies of both signs in two classes of their rest frames. The world lines of the bases of these reference frames at the origin extend into the upper and lower light cones.

At transforming from one $K_\pm$ to another inertial frame $K'_\pm$ in the same light cone, the coordinates and wave functions of a spinless particle are transformed as:

$$x_\pm^{\mu\,\prime} = \Lambda_{\nu\pm}^\mu x_\pm^\nu + b_\pm^\mu, \tag{41}$$

$$\psi_\pm'(x_\pm') = \sqrt{\frac{V}{V'}}\,\psi_\pm(\Lambda_{\nu\pm}^\mu x_\pm + b_\pm^\mu). \tag{42}$$

Here the matrices $\Lambda_{\nu\pm}^\mu$ form two orthochronous Lorentz groups $O_\pm^\uparrow$, including proper and improper Lorentz transformations in two classes of IFR without time axis inversion.

The simplest solutions of the wave equations are plane waves:

$$\psi_\pm(x_\pm) = \frac{1}{\sqrt{V}}u(p_\pm)e^{-ip_{\mu\pm}x_\pm^\mu/\hbar} \tag{43}$$

defined in two state spaces, where they are normalized to determine the probability for the corresponding particle type as in (10). They are written in coordinates within their light cones

and, in particular, $\psi_-(x_-)$ are written in coordinates K_-. However, to calculate the amplitudes of processes, measured quantities and wave functions must be assigned to only one type of frame, K_+, where observations are made, by transforming from K_- to K_+.

The transformation from K_- to K_+ at the initial moment $t_{0\pm}=0$ can be reduced to 4-inversion, where the coordinates and 4-momentum change sign. At other moments, $t_{0\pm}\neq 0$ the shift of the origin along time axis should be taken into account.

Notice that 4-inversion, as a "passive" transformation, does not change the world point (event) where the wave function is defined, but only expresses its coordinates in coordinates of a new reference frame. Therefore, two parts of the wave function with different energy signs should be written for the same world point $x_+^\mu=-x_-^\mu$. This means that if the wave function is defined at the point x_+^μ of K_+, then in K_- should be considered the wave function at the point $-x_-^\mu$. Then the expression $-ip_{\mu-}(-x_-^\mu)$ takes the form $ip_{\mu-}x_-^\mu$, and under 4-inversion, this scalar transforms to $ip_{\mu+}x_+^\mu$.

Thus, the free wave function $\psi_-(-x_-)$ under 4-inversion becomes equal to $\psi_-(x_+)$, where the difference from $\psi_+(x_+)$ is reduced to the difference in sign $p_\mu x^\mu$:

$$\psi_-(-x_-)\to\psi_-(x),\qquad \psi_\pm(x_\pm)\sim\frac{1}{\sqrt{V}}\exp(\mp ip_\mu x^\mu/\hbar). \tag{44}$$

Here, a variable without a subscript $\pm$ means that it is defined in K_+. As a result, the wave functions for two energy signs in K_+ have the form:

$$\psi(x,+E_p)=\frac{1}{\sqrt{V}}u(p)e^{-ip_\mu x^\mu/\hbar},\qquad \psi(x,-E_p)=\frac{1}{\sqrt{V}}v(p)e^{ip_\mu x^\mu/\hbar}. \tag{45}$$

At transforming from K_- to K_+ 4-vector j_-^μ, being a polar vector, changes sign. For free particles with $\psi_\pm(x_+)$ and 4-momenta $p_+^\mu=(\pm E_p/c,\vec{p}_\pm)$, the eigenvalues of the components of the probability current in (16), taking into account (13), do not depend on the values of $\bar{u}u$, $\bar{v}v$ and are equal to

$$j_{0\pm}=\pm\frac{1}{V_0}\frac{E_p}{mc},\qquad \vec{j}_\pm=\frac{1}{V_0}\frac{\vec{p}_\pm}{m}. \tag{46}$$

As we can see, the sign $j_\pm^0$ coincides with the sign of the energy $p^0=\pm E_p/c$. Introducing $V_0=VE_p/mc^2$ into (46), we obtain the usual relativistic expressions:

$$j_{0\pm}=\frac{\pm c}{V},\qquad \vec{j}_\pm=\frac{c^2}{V}\frac{\vec{p}_\pm}{E_p}=\frac{\vec{v}_\pm}{V}. \tag{47}$$

The considered transformations from K_- to K_+ were for vectors "passive", meaning that they don't change, but it changes the coordinate axes onto which they are projected. An "active" transformation can also be performed, remaining in K_+, where the coordinate axes are the same, but the sign of one or another vector changes. If this is a 4-momentum $p^\mu\to -p^\mu$, then such a transformation is a crossing transformation, and the first plane wave in (45) transforms into the second.

Thus, in the DKP formalism, the probability current for free particles, as it should be, coincides with the general probability current

3.3. General and special wave equations for gauge bosons

In the Standard Model, the only massless particles are photons and gluons, spin-1 gauge bosons, and, when gravity is included, also spin-2 gravitons. At considering *physical degrees of freedom*, they act as particles with helicity λ, which plays the role of a conserved chiral charge. If quanta with $\lambda > 0$ are considered as particles, then the same quanta with helicity $\lambda < 0$ behave as their charge-conjugate antiparticles.

Free gluons behave like eight types of photons, and only when gluons interact with each other and with quarks does their difference from photons become apparent. A similar situation is for free gravitons.

TS RQM, therefore, involves massless gauge bosons (photons and gluons with $\lambda = 1$, and gravitons with $\lambda = 2$) of positive (negative) energy that travel forward (backward) in time. The probability amplitudes of a free massless boson, from which their propagators are formed, satisfy the general wave equation (22) with $m = 0$:

$$\partial^{\mu}_{\pm}\partial_{\mu\pm}\psi^{\lambda}_{\pm} = 0, \quad \partial^{\mu}_{\pm}\partial_{\mu\pm}\psi^{\lambda+}_{\pm} = 0. \tag{48}$$

The DKP formalism for massless bosons is similar to that for finite-mass bosons, but contains additional constraints that do not fundamentally change the probability currents for free particles. Due to purely technical complications, this formalism will be discussed in the book [17].

A somewhat different and nontrivial picture appears in electroweak theory with nonzero mass vector bosons $W^{\pm}$. They arise in the theory as combinations of initially massless gauge bosons that acquire an effective mass through interaction with a background scalar field (condensate). Before acquiring mass W^{+}, and W^{-} had opposite helicities and charges, being a particle-antiparticle pair. Therefore, after acquiring mass W^{+}, and W^{-} will continue to have opposite spins and charges. Thus, in TS RQM, parity violation turns out to be related to the nature of $W^{\pm}$ as initially massless gauge bosons. Their longitudinal components, however, arise effectively as a result of interaction with the background field, replacing the three components of the scalar isodoublet.

The time-symmetric theory of interactions with gauge fields and gravitons within the framework of TS RQM will be considered in subsequent publications.

4. Time-symmetric theory of spin ½ fermions.

4.1. General and special wave equations for fermions

The boson theory of the former RQM with KG equation and corresponding probability current was, in principle, consistent. However, the negative sign of j^0 at negative energy was erroneously attributed to the probability itself, and due to this inadequate interpretation, the theory was considered as inconsistent. In TS RQM, the boson theory is consistent because the sign of j^0 is adequately interpreted as a current of positive probability in the negative direction of t axis.

Historically, the transition from the KG equation to the Dirac equation was explained by the need to ensure positivity of j^0 for negative energies as well, and this problem was considered as solved with this transition. But, ironically, this is precisely what made Dirac's theory untenable on this issue. In TS RQM, the sign of j^0 shows the direction of the probability

current along time axis for fermions also, because this follows from the general probability current, which is valid for free fermions also.

In this part of the article, the theory of spin ½ fermions based on TSRT will be presented, where the situation opposite to the case of the boson theory and the TS RQM leads for the first time to a completely consistent theory of fermions, introducing the necessary corrections to the previous Dirac theory.

For fermions of spin ½, as in the case of bosons, we present the standard relations with positive energy in K_+ and, due to invariance under 4-inversion, we write the same relations in K_- for particles evolving backward in ordinary time. The relationships in the two types of reference frames, due to their identity, are presented together in this case too.

The wave functions $\psi_\pm^r$ and $\overline{\psi}_\pm^r \equiv \psi_\pm^{r+}\gamma^0$ for particles of spin ½ in K_+ and in K_- are 4-row spinors for two values of spin ($r = 1,2$). The action functions and Lagrangians for them are:

$$S_\pm = \int d^4x \; \mathrm{L}_\pm, \quad \mathrm{L}_\pm = c\overline{\psi}_\pm(i\gamma^\mu\partial_{\mu\pm} - mc)\psi_\pm. \tag{49}$$

This yields the Dirac equation for spinors of both types, which is a special wave equation with a first time derivative:

$$(i\gamma^\mu\partial_{\mu\pm} - \kappa)\psi_\pm^r = 0, \quad \overline{\psi}_\pm^r(i\overleftarrow{\partial}_{\mu\pm}\gamma^\mu + \kappa) = 0. \tag{50}$$

This equation is covariant with respect to transformations (41) from $K_\pm$ to $K_\pm^{'}$ in the same light cone, as well as spatial reflections by 3-inversion matrix $P = diag(1,-1,-1,-1)$, which forms the orthochronous Lorentz group. The wave functions and Dirac matrices γ^μ are transformed through matrices $\tilde{S}(\Lambda_\pm)$, which, in addition to the usual definition, also include a volume factor:

$$\psi_\pm^{'}(x_\pm^{'}) = \tilde{S}(\Lambda_\pm)\psi_\pm(Px_\pm), \tag{51}$$

$$\tilde{S}^{-1}(\Lambda_\pm)\gamma^\mu\tilde{S}(\Lambda_\pm) = \Lambda_{\nu\pm}^\mu\gamma^\nu, \quad \tilde{S}(\Lambda_\pm) = \sqrt{\frac{V}{V'}}\exp\left(-\frac{i}{4}\omega_\pm\sigma_{\mu\nu}I_{n\pm}^{\mu\nu}\right)\tilde{P}. \tag{52}$$

where is $\tilde{P}$ the matrix of the corresponding representation of 3-inversion, $\omega_\pm$ is the "rotation angle" around the axis with direction n, $\sigma_{\mu\nu} = i[\gamma_\mu,\gamma_\nu]/2$ and $I_{n\pm}^{\mu\nu}$ is the rotation matrix. The relationship between the Hermitian conjugate and inverse matrices is:

$$\tilde{S}^{-1} = (V'/V)\gamma^0\tilde{S}^+\gamma^0 = (E_p/E_{p'})\gamma^0\tilde{S}^+\gamma^0. \tag{53}$$

The simplest solutions of the equations (50) are plane waves:

$$\psi_\pm^r(x_\pm) = \frac{1}{\sqrt{V}}u_{p_\pm}^r\exp(\mp ip_{\mu\pm}x_\pm^\mu/\hbar). \tag{54}$$

Spinors $u_{p_\pm}^r$ are easiest to find in rest frames of particles with two energy signs $K_\pm^{(0)}$. In these frames, the Eq. (50) simplify and yield a wave function:

$$i\gamma^0\partial_{0\pm}\psi_{0\pm}^r = \kappa\psi_{0\pm}^r, \quad i\,\partial_{0\pm}\overline{\psi}_{0\pm}^r\gamma^0 = -\kappa\overline{\psi}_{0\pm}^r, \tag{55}$$

$$\psi_{0\pm}^r = \frac{1}{\sqrt{V_0}}u_{0\pm}^r\exp(-imc^2t_\pm/\hbar), \tag{56}$$

$$u^1_{0\pm} = \begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix}^T, \quad u^2_{0\pm} = \begin{pmatrix}0 & 1 & 0 & 0\end{pmatrix}^T. \tag{57}$$

Next, using (51), we transform the wave functions (57) from $K^{(0)}_{\pm}$ into $K_{\pm}$:

$$\psi^r_{\pm}(x) = \frac{1}{\sqrt{V}} u^r_{p_\pm} \exp(\mp i p_\mu x^\mu / \hbar). \tag{58}$$

Here the spinors $u^r_{p_\pm}$ from (54) take the form [5,6]:

$$u^1_{p_\pm} = A_\pm \begin{pmatrix} 1 \\ 0 \\ p_{z\pm} / \tilde{p}_\pm \\ (p_{1\pm} + i p_{2\pm}) / \tilde{p}_\pm \end{pmatrix}, \quad u^2_{p_\pm} = A_\pm \begin{pmatrix} 0 \\ 1 \\ (p_{1\pm} - i p_{2\pm}) / \tilde{p}_\pm \\ -p_{z\pm} / \tilde{p}_\pm \end{pmatrix}, \tag{59}$$

where $\tilde{p}_\pm = mc + E_{p\pm} / c$ and $A_\pm = \sqrt{\tilde{p}_\pm / 2mc}$. This gives the normalization conditions:

$$\int_V d^3x \psi^{r+}_{p_\pm} \psi^{r'}_{p_\pm} = \frac{E_{p\pm}}{mc^2} \delta^{rr'}, \qquad \int_V d^3x \bar{\psi}^r_{p_\pm} \psi^{r'}_{p_\pm} = \delta^{rr'}. \tag{60}$$

Note again that the definition $V = 1$ in all reference frames does not take into account the fact that the bilinear product of wave functions is the probability density. In relativistic theory, only the proper normalization volumes V_0 are invariant, and only the normalization $V_0 = 1$ is natural, which leads to $V = mc^2 / E_p$.

The eigenvalues f of the operator F, as the relation (13) shows, are determined by dividing the matrix element by the normalization factor $\bar{\psi}\psi$. For negative energy fermions the form $\bar{\psi}\psi$ is non-trivial, since at transforming from K_- to K_+, $\bar{\psi}$ changes the sign. Therefore, the representation (13) allows one to determine the eigenvalues of the operators of physical quantities regardless of this specific property of spinors.

4.2. Transformation of wave functions from a reference frame going backward in time to a usual frame

Events described in two types of reference frames $K_\pm$ are related by 4-inversion, which changes the signs of the projections of polar 4-vectors, in particular, momentum $p^\mu_- \to -p^\mu_+$, the probability current density $j^\mu_- \to -j^\mu_+$ and the interaction currents. The transformation matrices of 4-vectors under inversion of time axis T_v, the spatial coordinate axes P_v, and their product, 4-inversion $P_v T_v$, have the form:

$$(P_v)_{\mu\nu} = -(T_v)_{\mu\nu} = diag(1,-1,-1,-1) = g_{\mu\nu}, \quad P_v T_v = -I, \tag{61}$$

where I is the identity matrix, $g_{\mu\nu}$ is the metric tensor in Cartesian coordinates in Minkowski space. We now consider the matrices of spinor transformations under coordinate inversion. P_s, T_s and 4-inversions $P_s T_s$. The Dirac equation (50) is covariant under 4-inversion of coordinates $\partial / \partial x_\mu \to -\partial / \partial x_{\mu-}$ if the matrices $P_s T_s$ anticommute with γ^μ, are equal to γ^5 (up to a phase factor, taken equal to 1):

$$(P_s T_s)^{-1} \gamma^\mu P_s T_s = -\gamma^\mu, \quad P_s T_s = i\gamma^0\gamma^1\gamma^2\gamma^3 = \gamma^5. \tag{62}$$

The transformation matrix of spinors $\tilde{S}_4(\Lambda_\pm)$ from the general Lorentz group includes, in addition to the matrix $\tilde{S}(\Lambda_\pm)$ from (52), 4-inversion matrix γ^5:

$$\psi'_\mp(x'_\mp) = \tilde{S}_4(\Lambda_\pm)\psi_\pm(-x_\pm), \tag{63}$$

$$\tilde{S}_4(\Lambda_\pm) = \gamma^5\tilde{S}(\Lambda_\pm) = \gamma^5\sqrt{\frac{V}{V'}}\exp\left(-\frac{i}{4}\omega_\pm\sigma_{\mu\nu}I^{\mu\nu}_{n\pm}\right), \tag{64}$$

$$\tilde{S}_4^{-1}(\Lambda_\pm)\gamma^\mu\tilde{S}_4(\Lambda_\pm) = -\Lambda^\mu_{\nu\pm}\gamma^\nu. \tag{65}$$

The transformation matrix of spinors under inversion spatial coordinates is known: $P_s = \gamma^0$. This allows us to find from (62) the spinor transformation matrix T_s at reflecting time axis (in standard representation):

$$T_s = \gamma^0\gamma^5 = i\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad T_s^{-1} = T_s^+ = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} = -T_s, \quad T_sT_s = -I. \tag{66}$$

As we can see, a double time inversion, according to (66), changes the sign of the wave function.

Next, we take into account that the two parts of the wave function with opposite energy signs are written in K_+ for a common world point x^μ_+. Therefore, as in the case of bosons, in terms of coordinates K_-, the wave function should be taken at the point $-x^\mu_-$, i.e. $\psi_-(-x_-)$.

In the former standard formulation of RQM, the time axis inversion operation (66) was separated into two operations, i.e.:

a) *charge conjugation* $\psi_c = i\gamma^2\psi^*$, consisting of complex conjugation and multiplication by $C = i\gamma^2$, and

b) *Wigner time reversal*, which involves complex conjugation with permutation of the initial and final states, and multiplication to the matrix $T_W = i\gamma^1\gamma^3$.

At applied together and with the addition of inversion $t \to -t$, we obtain:

$$iT_W[C\psi^*(-t)]^* = -\gamma^1\gamma^3(i\gamma^2)^*\psi(-t) = i\gamma^1\gamma^2\gamma^3\psi(-t) = T_s\psi(-t), \tag{67}$$

and as a result, time axis inversion operator turns out to be the same as in (66):

$$iCT_W = i\cdot i\gamma^2\cdot i\gamma^1\gamma^3 = i\gamma^1\gamma^2\gamma^3 = T_s. \tag{68}$$

Under 4-inversion (62), the spinors for a free particle $\bar{u}^r_{p_-}$, $u^r_{p_-}$ transform as:

$$P_sT_su^r_{-p_-} = \gamma^5u^r_{-p_-} = v^r_{p_+}, \tag{69}$$

$$\bar{u}^r_{-p_-}(P_sT_s)^+ = u^{r+}_{-p_-}\gamma^0\gamma^5 = -u^{r+}_{-p_-}\gamma^5\gamma^0 = -\bar{v}^r_{p_+}, \tag{70}$$

where $\mathbf{p}_+ = -\mathbf{p}_-$ is a 3-momentum in K_+. In detail, the transformations (69) have the form:

$$\gamma^5u^1_{-p_-} = A_-\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ -p_{3-}/\tilde{p} \\ -(p_{1-}+ip_{2-})/\tilde{p} \end{pmatrix} = A_-\begin{pmatrix} p_{3+}/\tilde{p} \\ (p_{1+}+ip_{2+})/\tilde{p} \\ 1 \\ 0 \end{pmatrix} = v^1_{p_+}, \tag{71}$$

$$\gamma^5 u^2_{-p_-} = A_- \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -(p_{1-} - ip_{2-})/\tilde{p} \\ p_{3-}/\tilde{p} \end{pmatrix} = A_- \begin{pmatrix} (p_{1+} - ip_{2+})/\tilde{p} \\ -p_{3+}/\tilde{p} \\ 0 \\ 1 \end{pmatrix} = v^2_{p_+}. \quad (72)$$

As we can see, 4-inversion does not change the helicity of the particle.

So, for a plane wave (54) we have $-ip_{\mu-}(-x^\mu_-) = ip_{\mu+}x^\mu_+$ and therefore:

$$\psi^r_-(x) = P_s T_s \psi^r_{p_-}(-x_-) = \frac{1}{\sqrt{V}} v^r_{p_+} \exp(ip_{\mu+}x^\mu_+/\hbar). \quad (73)$$

Transformation $\bar{\psi} = \psi^+\gamma^0$:

$$\bar{\psi}_-(x) = \psi^+_{p_-}(-x_-)\gamma^0\gamma^5 = -\psi^+_{p_-}(-x_-)\gamma^5\gamma^0, \quad (74)$$

gives a result that differs in sign from the Hermitian conjugate (73):

$$\bar{\psi}_-(x) = \frac{1}{\sqrt{V}}(-\bar{v}_p)\exp(-ip_\mu x^\mu/\hbar). \quad (75)$$

As the result, for the negative energy states the bilinear form $\bar{\psi}_-\psi_-$ in K_+ is expressed through the form $\bar{\psi}\psi$ from K_- with a minus sign: $\bar{\psi}_-\psi_- = -\bar{\psi}_+\psi_+ \sim \bar{v}v$.

The eigenvalues f_{PT} of the operators of physical quantities F are found from the matrix element of this operator from (13). For states of negative energy, after 4-inversion $F_{PT}(x) = \gamma^5 F(-x)\gamma^5$, we obtain:

$$f_{PT} = \frac{\bar{\psi}_- F_{PT}(x)\psi_-}{\bar{\psi}_-\psi_-}, \quad (76)$$

Since for the negative energy fermions the form $\bar{\psi}\psi$ changes sign at transforming from K_- to K_+, then the representation (76) allows us to find f_{PT} regardless of this specificity and normalization factors of the wave functions of fermions.

In Dirac's theory, the inclusion of a part of the improper transformations, the inversion of the axes of spatial coordinates, led to the unification into one spinor of two spinors in the chiral representation, the "left" and "right" spinors, which transform into each other under such a spatial inversion.

Improper transformations in the general Lorentz group include, in addition to the inversion of the spatial coordinate axes, also the inversion of time axis, and therefore its representations are two-valued and there are two types of spinors u^r_p and v^r_p, which differ in the sign of energy. But there is also a single-valued representation when two spinors are combined into one tetraspinor [7]. The tetraspinor representation is briefly discussed in Appendix 8.1 and in more detail in [17].

4.3. Lagrangian, probability current, and interaction currents for negative energy fermions in the usual reference frame

In Dirac's theory, the probability current density has the form:

$$j^\mu_D = c\bar{\psi}\gamma^\mu\psi. \quad (77)$$

The Gordon expansion for free particles has no spin part and simplifies, allowing the probability current to be expressed in terms of 4-momentum eigenvalues p^μ:

$$j_D^\mu = \frac{p^\mu}{m}\bar{\psi}\psi. \tag{78}$$

At negative energy $p^0 < 0$ the bilinear form $\bar{\psi}\psi \sim \bar{\nu}\nu < 0$ is negative, and the time component j_D^0 appears as positive. This was considered as an achievement of Dirac's theory, mistakenly believed to j_D^0 yield the probability density directly. Furthermore, the spatial components of the probability current were inverse to the momentum $j_D^i \sim -p^i$. From a physical perspective, these two properties were absurd, since in relativistic kinematics, the sign of the probability current j^μ must coincide with the sign of p^μ. For bosons, this requirement of relativistic kinematics is fulfilled, and therefore these consequences of Dirac's theory testified to the incompleteness of the theory of fermions.

In TS RQM, these problems of Dirac's fermion theory are absent, and the probability current satisfies the stated requirement. This difference from the former theory follows from the fact that the current, instead of $\bar{\psi}\psi$ in (78), which is negative at negative energy, includes $\bar{\psi}_0\psi_0 > 0$, just like in (16). Moreover, the part of the interaction currents that corresponds to identical momenta of the initial and final spinors is the product of the probability current and the charge and also changes sign under 4-inversion.

The total probability current j^μ (16) for free fermions is defined as the product of the drift velocity, proportional to $\bar{\psi}\ddot{\partial}^\mu\psi / \bar{\psi}\psi$, and the invariant probability density in the rest frame $\bar{\psi}_0\psi_0 > 0$:

$$j^\mu = \frac{i\hbar}{2}\frac{\bar{\psi}\partial^\mu\psi - (\partial^\mu\bar{\psi})\psi}{\bar{\psi}\psi}\bar{\psi}_0\psi_0 = \frac{p^\mu}{m}\bar{\psi}_0\psi_0 \sim p^\mu. \tag{79}$$

Here ∂^μ is a diagonal operator relating the same component of the wave function in the initial and final states and j^μ is therefore also a diagonal operator. Since $\bar{\psi}_0\psi_0 > 0$, then probability current has the same sign as 4-momentum and changes sign upon 4-inversion.

In this case, for a free particle, the total probability current density coincides with the correct current density that follows from the Dirac equation, taking into account the definition of this current similarly (79) as:

$$j^\mu = \frac{c\bar{\psi}\gamma^\mu\psi}{\bar{\psi}\psi}\bar{\psi}_0\psi_0 = \frac{p^\mu}{m}\bar{\psi}_0\psi_0 \sim p^\mu. \tag{80}$$

The probability current density for a negative energy particle follows from the Lagrangian taking into account the volume factor and 4-inversion of spinors:

$$j_-^\mu = \frac{\delta L_-}{\delta(\partial_\mu\psi_-)}(-i\psi_-) = c\bar{\psi}_-\gamma^\mu\psi_-\frac{V}{V_0} \sim c\bar{\nu}\gamma^\mu\nu\frac{V}{V_0}. \tag{81}$$

Using the Gordon expansion, we express the probability current through the eigenvalues of 4-momentum and obtain:

$$j^\mu = c\bar{\psi}_-\gamma^\mu\psi_-\frac{V}{V_0} = \frac{p^\mu}{m}\frac{V}{V_0}\bar{\nu}\nu = \frac{p^\mu}{m}\frac{V}{V_0}. \tag{82}$$

Thus, in the theory of fermions, the signs of the probability current and 4-momentum are the same and the requirements of TSRT are satisfied.

The interaction current density in a vector field A_μ follows from the Lagrangian taking into account the volume factor and 4-inversion of spinors:

$$j^\mu_{e-} = \frac{\delta L_-}{\delta A_\mu} = ce\overline{\psi}_-\gamma^\mu\psi_-\frac{V}{V_0} \sim ce\overline{v}\gamma^\mu v\frac{V}{V_0}. \tag{83}$$

Expressing the current through the eigenvalues of 4-momentum, we obtain:

$$j^\mu_{e-}(p',p) = \frac{e}{2m}\overline{v}(p')\left[(p'^\mu + p^\mu) + i(p'^\mu - p^\mu)\sigma^{\mu\nu}\right]v(p) \tag{84}$$

Thus, in the theory of fermions, the signs of the current and 4-momentum are the same for the value $e>0$ and $p'^\mu \simeq p^\mu$, satisfying the requirements of TSRT.

The presence of spin should not lead to a fundamental difference between free bosons and fermions. Therefore, the proportionality of the probability current vector to the energy-momentum vector is natural and consistent with TSRT. Spin also plays no role in single-particle problems with scalar potentials.

The presence of a spin component in currents interacting with vector or tensor fields is also natural, yielding results consistent with experiment. However, this is only indirectly related to the probability current, which expresses the properties of free particles independent of any interactions.

4.4. Exclusion of paradoxes of fermion theory in TS RQM

Two paradoxical consequences of Dirac's theory from a physical point of view were the velocity paradox and high-frequency fermion oscillations (or zitterbewegung). In TS RQM, the probability current density of free fermions is proportional only to the energy-momentum, and it does not have these two paradoxes.

The velocity paradox was that in Dirac's theory the value of the 3-velocity of fermions appeared equal to the speed of light. In TS RQM, this paradox disappears because 4-momentum enters into expressions for the probability current exactly as in relativistic theory. Clearly, in the limit of nonrelativistic quantum mechanics and classical relativistic mechanics, this current transforms into expressions with the usual 3-velocity.

In the zitterbewegung paradox, Dirac's theory predicted transitions between states of two energy signs in wave packets $\overline{u}^r_p\gamma^i v^{r'}_{p'}$ and $\overline{u}^r_p\gamma^i v^{r'}_{p'}$ at localization of free fermions in regions of the order of the Compton wavelength [5,6]. The contradiction of this statement consisted of three circumstances.

In TS RQM, the probability current density of a free fermion (79) includes only 4-momentum, which is an even operator that does not mix states of opposite energy signs. There is no spin component, which previously led to such mixing and zitterbewegung. This means that in TS RQM, in agreement with QFT, there is no zitterbewegung of free fermions. The influence of the potential barrier confining the particle's wave packet in a finite volume leads to pair production, but this is a result of interaction with the confining potential and is unrelated to the states of a free particle.

Another paradox of fermion theory, the Klein paradox, is known to be eliminated by taking into account pair production in a strong field, i.e., beyond the single-particle approximation. Therefore, this effect is paradoxical only within the single-particle approximation and becomes a normal pair production effect under a more precise analysis. TS RQM, based on ZSF interpretation, is a many-particle approach and naturally includes pair production and annihilation. Therefore, in it, this process ceases to be a paradox, as will be shown in Section 5.1.

Thus, TS RQM brings the theory of fermions into agreement with the relativistic theory, and also eliminates the former paradoxes.

5. Description of interactions of particles in TS RQM

5.1. Solutions to the Klein paradox in TS RQM and QFT

RQM was created on the basis of the single-particle approximation. The well-known Klein paradox, which arose at the very beginning of RQM, was considered a demonstration of RQM's inadequacy for describing particle interaction processes requiring a multiparticle approach, and therefore it was believed that this paradox could only be resolved within the framework of QFT, taking into account pair production in a strong field [5,6].

However, with the discovering of ZSF interpretation and the propagator approach, RQM has become, in fact, the same many-particle theory as QFT, reproducing the diagram technique and allowing the description of practically all particle processes, including pair production. The Klein paradox demonstrates that a paradoxical phenomenon in the single-particle approach becomes ordinary and understandable within RQM framework when the multiparticle nature of the process is taken into account in simplest way, without complexities of QFT. Here we present a brief description of the solution, and the details are given in the book [17]. Starting from this section, the system of units $c=1$ and $\hbar=1$ will be used.

Let a spinless particle of energy E move freely along the x axis at $x<0$, but at $x\geq 0$ there is a potential step of height V. In a weak field, $V<E-m$ there are transmitted and reflected parts of the initial current, while in a moderate field, $E-m<V<E+m$ the current is completely reflected. The processes in these regimes are the same as in nonrelativistic theory.

In the third, strong-field regime $V>E+m$ the former RQM with the single-particle approximation led to the Klein paradox: only one current of initial particles was considered, which at the barrier split into two parts, moving away from the barrier in opposite directions. The current outgoing from the barrier turned out to be greater than the incident current, and a negative current also appeared in the region of the step [5,6].

In TS RQM, as well as in the QFT, the solution to the paradox is clear from a physical point of view: at $V>E+m$ the initial current is completely reflected, but a new current of particle-antiparticle pairs generated at the barrier appears, which go in opposite directions.

In QFT, a dynamic approach is needed to describe processes in the strong field regime, since the total wave function in each region includes the wave functions of particles and antiparticles included in the initial, reflected or created currents. In RQM, the picture is simplified, since the initial current is completely reflected, and an additional flow of negative energy particles comes from the future, moving from right to left, is reflected on the barrier along the time axis, also changing the sign of the energy, and goes back into the future.

Thus, let's compare three approaches to analysis of the process:

- in the former RQM, only *one* initial current was considered, which was divided at the barrier into two parts - reflected and transmitted, and a paradox arose: the reflected current turned out to be greater than the initial one;
- In TS RQM, *two current*s are considered: the initial one, which is completely reflected on the barrier, and the current of negative energy particles from the right, which changes the sign of energy on the barrier, reflecting in time, and continues to go further to the left;
- In QFT, *three current*s are described: the initial one, completely reflected on the barrier, and two currents of pairs created on the barrier- created particles going to the left, and corresponding current of created antiparticles going to the right.

The last two are physically correct and do not lead to paradoxes. They are essentially equivalent, as they can be reduced to the same diagrams. However, their treatment in TS RQM is much simpler. This is due to the property of TS RQM that particle and antiparticle currents are considered as two parts of a single current, whose energy and time evolution direction signs change at interaction with the barrier.

5.2. Causal propagators

In TS RQM, causal propagators arise naturally from time-symmetric solutions of the wave equations in the presence of interactions. For a spinless particle, the causal propagator G_c is the Green's function under the ZSF boundary conditions in K_+, corresponding to the forward in time propagation of positive-energy particles and the backward in time propagation of negative-energy particles.

For KG equation with a source represented by the interaction current $j(x)$:

$$(\partial_\mu \partial^\mu + m^2)\phi(x) = j(x), \tag{85}$$

the propagators $G_{c\pm}(x'-x)$ are defined for solutions of two energy signs as:

$$(\partial_\mu \partial^\mu + m^2)G_{c\pm}(x'-x) = \delta^4(x'-x). \tag{86}$$

The solution of (85) in ordinary time at the ZSF boundary conditions has the form:

$$\phi(x') = \phi^{(0)}(x') + \int_{-\infty}^{\infty} dt \int d^3x\, G_{c+}(x'-x)\, j(x) + \int_{\infty}^{-\infty} dt \int d^3x\, G_{c-}(x'-x)\, j(x). \tag{87}$$

Here $\phi^{(0)}$ is a solution of the homogeneous equation with the same boundary conditions, $G_{c+}(x'-x) \sim \theta(t'-t)$ describes the transfer of positive energy forward in time, and $G_{c-}(x'-x) \sim \theta(t-t')$ - transfer of negative energy backward in time.

Since the limits of the second time integral in (87) are rearranged, we obtain the full causal propagator G_c by rearranging the limits of the second integral, which changes the sign in front of G_{c-}, and Eq. (87) takes on a more compact form:

$$\begin{aligned} \tilde{\phi}(x) &= \tilde{\phi}^{(0)}(x) + \int_{-\infty}^{\infty} dt' \int d^3x' [G_{c+}(x-x') - G_{c-}(x-x')]\, j(x') = \\ &= \tilde{\phi}^{(0)}(x) + \int_{-\infty}^{\infty} dt' \int d^3x' G_c(x-x')\, j(x'), \end{aligned} \tag{88}$$

$$G_c(x) = G_{c+}(x) - G_{c-}(x). \tag{89}$$

Thus, at expressing the inverse time integral in terms of the forward time integral, the causal propagator G_c is given *by the difference* of G_{c+} and G_{c-}. Its momentum representation has the standard form:

$$G_c(x'-x) = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x'-x)}}{p^2 - m^2c^2 + i\varepsilon}. \tag{90}$$

The causal propagator $S_c(x'-x)$ for particles with spins 0, 1 and ½ satisfies the special wave equation with a point source:

$$(i\hbar\Gamma^\mu \partial_\mu - mc)S_c(x'-x) = \delta^4(x'-x), \tag{91}$$

and, at the ZSF boundary conditions, similarly (88) and (89), takes the standard form:

$$S_c(x'-x) = \int \frac{d^4p}{(2\pi)^4} \frac{\Gamma^\mu p_\mu + mc}{p^2 - m^2c^2 + i\varepsilon} e^{-ip(x'-x)}. \tag{92}$$

Both types of propagators (90) and (92) give non-zero transition probabilities for positive energy at $t' > t$ and for negative energy at $t' < t$.

5.3. Scattering matrix and diagram technique

In TS RQM, the diagram technique at the presence of interactions gives practically the same final results as the former RQM [5,6], but in a more natural way, i.e. without the former problems and artificial rules.

Since its foundation in the 1940s, it has been known that the formalism of scattering theory in RQM becomes compact and covariant precisely in terms of particles of two energy signs evolving in opposite directions of time axis. TS RQM reproduces this formalism of covariant scattering theory, and the reasons for this are the following:

- the causal propagators underlying the covariant diagram technique follow from the time-symmetric expressions of TS RQM;
- matrix elements with antiparticles are described by expressions with negative energy particles, which are obtained by 4-inversion of the states in K_- or by crossing-transformation of positive energy states in K_+.

The standard *S*-matrix, which was also used in QFT, was defined as the transition amplitude from states in the infinite past to states in the infinite future. Its nontrivial part, in the case of a single vertex, is the product of two transition amplitudes - from states in the past to the interaction point at the finite moment $-\infty < t < \infty$ and from states at this moment to states in the future:

$$S_{+-} = \langle \psi(+\infty) | \psi(t) \rangle \langle \psi(t) | \psi(-\infty) \rangle. \tag{93}$$

An example of these processes is the potential scattering of a positive energy particle.

Other components of *the S-matrix also appear* in RQM. These are state transitions:

- from the future to states in the past through a finite moment t:

$$S_{-+} = \langle \psi(-\infty) | \psi(t) \rangle \langle \psi(t) | \psi(+\infty) \rangle, \tag{94}$$

in particular, scattering of negative energy particles on the potential;

- from the future to states also in the future:

$$S_{++} = \langle \psi(+\infty) | \psi(t) \rangle \langle \psi(t) | \psi(+\infty) \rangle, \tag{95}$$

in particular, the pair creation at the potential;

- from the past to states also in the past:

$$S_{--} = \langle \psi(-\infty) | \psi(t) \rangle \langle \psi(t) | \psi(-\infty) \rangle, \tag{96}$$

in particular, the annihilation of a pair at the potential.

At considering the simple processes with interaction vertices but without loop diagrams, i.e., tree diagrams, two facts become clear, which are also typical for loop diagrams. First, the results of former calculations are reproduced; second, the TS RQM leads to a more compact formalism due to the time-symmetric form of the expressions for particles of both energy signs.

The diagrams with positive energy particles are obviously the same as in the former RQM, and the diagrams with negative energy particles are obtained from them by:

- 4- inversion, when the signs of the energy of all particles change, and
- crossing transformation, when the signs of the energy of some of particles change.

These issues are discussed in more detail in the book [17].

Thus, TS RQM is, from a formal point of view, a natural theoretical basis for the heuristic formalism of covariant scattering theory.

6. Gravitational self-regularization of loop diagrams at Planck scales

6.1. The absence of ultraviolet divergences at taking into account the external gravitational field of system of particles in loop diagram

In the former relativistic quantum theory, including propagator approach, transition amplitudes were generally considered in Minkowski space. In this case, loop diagrams in higher orders of perturbation theory contained ultraviolet divergences and, in fact, the theory became inconsistent. Renormalization, with the replacement of divergent masses and charges with their observed values, allowed the problem to be circumvented in practice, but did not solve it. This fundamental problem of the theory remained unsolved until recently [5,6].

In flat spacetime, loop diagrams of gauge fields interacting with fermions grow logarithmically $a\ln(\Lambda/m)$ with the cutoff energy Λ, where m is the mass of a light fermion. For SM fields the coefficients a are small $a \ll 1$ up to the Planck energy $\Lambda_g \sim 10^{19}$ GeV. Further extrapolation to higher cutoff energies $\Lambda > \Lambda_g$, without taking into account the general relativity restrictions, then led to divergences. At including gravity in the form of graviton exchange on a flat background, estimates showed that the contribution of gravitons to L-loop diagrams with increases with Λ as $\sim(\Lambda/\Lambda_g)^{2L+2}$, i.e., one-loop diagrams grow as, $\sim(\Lambda/\Lambda_g)^4$ and then each additional loop adds two to the growth rate. The graviton contribution therefore remains small at $\Lambda < \Lambda_g$, but diverges catastrophically at $\Lambda \geq \Lambda_g$.

In fact, it turned out that the problem of ultraviolet divergences, like the problem of zero-point energy, can be solved without new hypotheses by consistently taking into account the requirements of relativistic theory, in this case general relativity [14]. The physical mechanism leading to the finiteness of the perturbation theory series turned out to be the external gravitational field of the system of particles in the loop diagram and the gravitational dilation of the proper times of the particles in this field with freezing at Planck distances l_g.

In general relativity there are three types of time dilation:

1) *Inertial time dilation* is relative and symmetrical for two clocks at rest in two inertial frames.
2) *Non-inertial time dilation*, the dilation of the proper time of a non-inertial clock relative to the proper time of an inertial clock, is absolute and asymmetric for the two clocks, as demonstrated by the clock (twin) paradox. At comparing the proper times of two differently accelerated clocks, it will lag more a clock accelerated stronger and/or longer.
3) *Gravitational time dilation* is the slowing down of the proper times of standard clocks at rest closer to the gravitational source, compared to the proper times of distant clocks. The equivalence principle implies that gravitational time dilation is similar to non-inertial one and is also absolute and asymmetric for these two clocks.

An external gravitational field generated by the total energy of a system of particles in a loop diagram slows down the proper times of these particles relative to the time t of the global hypersurface of simultaneity, where the *S*-matrix is defined. This time dilation is irreversible and absolute at comparing the times of two mutually at rest clocks, i.e., the distant clock and the proper "clock" in the system of particles.

To understand the failure of previous attempts to account for gravity in quantum theory, it's enough to recall a similar story from atomic physics, where the electromagnetic field had to be taken into account. In that case, accounting for the non-quantized part of the field, the Coulomb interaction, made it possible to explain the discreteness of energy levels and the entire structure of atoms and molecules. Including the quantized part, photon exchange, only slightly shifted the Coulomb levels (the Lamb shift). If we had followed the same approach as with

gravity, "forgetting" the Coulomb field and attempting to explain the structure of atoms solely by photon exchange, then they wouldn't be able to explain or describe anything.

But this is precisely what happened with gravity: ignoring the contribution of the non-quantized part of the field, the time component of the metric, leading to gravitational regularization due to the freezing of all processes at the Planck scale, they took into account only the contribution of the quantized part of the metric without the main effect - gravitational freezing and obtained a catastrophic increase in high-energy contributions, i.e. divergences.

Appendix 8.2 provides some details of the gravitational freezing of processes and gravitational self-regularization at Planck scales. Below we consider the gravitational regularization of known interactions in the simplest case, when it is reduced to a cutoff at the invariant Planck energy.

6.2. Gravitationally regularized SM and quantum gravity.

In this section we present the results of gravitational regularization within the framework of the standard diagram technique.

In quantum electrodynamics (QED), the constants Z_2 and Z_3 are the renormalization constants of the electron (S_c) and photon (G_c) propagators, respectively: $S_c = Z_2^{-1} S_c^{(0)}$ and $G_c = Z_3^{-1} G_c^{(0)}$. The renormalized propagators are expressed in terms of their proper energy diagrams. For the photon, such a diagram is given by the transverse part of the polarization operator $\Pi_{ik}(q^2) = (q_i q_k - q^2 g_{ik})\Pi(q^2)$. The correction to the electron mass is released from its proper energy $\Sigma(p)$, where $\Sigma(p) = [\Delta m - (Z_2^{-1} - 1) + C(p)](p - m).$ and $S_c^{-1} = (S_c^{(0)})^{-1} - \Sigma$. From the Ward identity $\partial\Sigma(p)/\partial p^i = -\Lambda_i(p, p)$, where $\Lambda_i(p, p')$ is the contribution of the vertex part, we have $Z_1 = Z_2$ and for the physical charge we obtain $e = e_0 Z_1 Z_2^{-1} Z_3^{1/2} = e_0 Z_3^{1/2}$ [5].

In its simplest form, gravitational self-regularization of loop diagrams reduces to cutting of momentum integrals at the invariant Planck energy Λ_g. In QED with electrons and photons, the largest contribution to the renormalization constants Z_1, Z_2, Z_3 comes from one-loop diagrams with electrons in an intermediate state, which include the coupling constant $\alpha/4\pi \simeq 0.00058$ and the factors:

$$\ln\frac{\Lambda_g^2}{m_e^2} \simeq 103.056, \quad A_e = \frac{\alpha}{4\pi}\ln\frac{\Lambda_g^2}{m_e^2} \simeq 0.0598\,. \tag{97}$$

The polarization operator $\Pi(q^2)$ (at $q^2 \to 0$) and Z_3, the renormalization constant of the photon propagator, have the values:

$$\Pi(0) \simeq \frac{4}{3}A_e \simeq 0.0797 \simeq 7.97\%, \quad Z_3 \simeq 1 - \Pi(0) = 0.9203. \tag{98}$$

They give for the initial coupling constant α_0 and corrections $\Delta(e^2)$ to e^2:

$$\alpha = Z_3\alpha_0, \quad \alpha_0 \simeq \frac{\alpha}{0.92} \simeq \frac{1}{126.07} \simeq 0.0079, \tag{99}$$

$$\Delta(e^2) = \Pi(0)e^2 = 0.0797\,e^2. \tag{100}$$

The other two renormalization constants have the values:

$$Z_1^{(1)} = Z_2^{(1)} \simeq (1+A_e)^{-1} \simeq 0.9436. \quad (101)$$

For an electron, the bare mass $m_{e(0)}$ is then equal to about 0.85 of the physical mass m_e, and the difference between them is about $0.15m_e$:

$$m_{e(0)} = m_e(1+3A_e^{(1)})^{-1} \simeq 0.8479m_e = 0.4333\,\text{MeV}, \quad (102)$$

$$\Delta m_e = m_e - m_{e(0)} \simeq 0.1521m_{e(0)} = 0.0777\,\text{MeV}. \quad (103)$$

Higher-order loop contributions to QED also remain small, and in all cases, gravitational regularization leads to sufficiently small corrections to the observables. Calculations of constants $Z_i^{(n)}$ for n-loop diagrams using various methods show that in QED perturbation theory converges.

In the SM, based on the symmetry group $SU(3)\times SU(2)\times U(1)$ and spontaneous symmetry breaking with a scalar condensate, loop diagrams are finite at regularization energy Λ_g. For non-Abelian gauge fields, they are small due to the logarithmic dependence on Λ_g and asymptotic freedom. In electroweak theory and QCD, loop corrections reduce to modifications of the effective constants by logarithmic factors. At ordinary distances, their coupling constants are larger than α, but decrease at small distances and l_g become smaller near α. As a result, the perturbation series converge.

The contributions of triangular diagrams with fermionic propagators and a pseudovector vertex increase linearly with the cutoff energy Λ and lead to chiral anomalies. Under gravitational regularization, the contribution of such diagrams is finite, but of the order of $\sim\Lambda_g$. Since such large contributions are unacceptable, the criterion for the consistency of particle physics models remains the mutual cancellation of chiral anomalies, which is what occurs in the SM. Gravitational regularization makes the situation with chiral anomalies in the SM rational, since finite quantities cancel each other.

Regularization of loop contributions by the self external gravitational field through a metric g_{00} with a cutoff at Λ_g also occurs in quantum gravity in the propagator approach to perturbation theory. Interactions through the transverse components of the gravitational field are described by the corresponding propagators. Representing gravitons as special particles, we conclude that their presence in the system of source particles enhances the gravitational dilation of proper times.

The dimensional coupling constant in quantum gravity $G\sim 1/\Lambda_g^2$ leads to power-law contributions from loop diagrams with gravitons $\sim(\Lambda/\Lambda_g)^{2L+2}$. However, unlike a scalar field, the effective coupling constant $(\Lambda/\Lambda_g)^2<1$ is small up to Λ_g. One-loop contributions of order $(\Lambda/\Lambda_g)^4$ are small for $\Lambda<\Lambda_g$, and contributions from multi-loop diagrams, where each loop adds two to the growth rate, can be neglected. Thus, perturbation theory remains valid for $\Lambda<\Lambda_g$, quantum gravity turns out to be finite, and perturbation theory apparently converges.

Thus, gravitational regularization following from gravitational time dilation leads to a finite limit for the regularization energy $\Lambda\to\Lambda_g$, which turns out to be sufficient for the finiteness and renormalizability of the SM gauge fields and quantum gravity.

In this situation, the only problematic factor is the scalar sector introduced for spontaneous symmetry breaking. Loop integrals for the scalar field grow as Λ^n with $n>1$,

leading to finite but unacceptably large contributions to observables of the order Λ_g^n. Therefore, either the scalar particle is composite with an effective field, or SM must employ a different mass generation mechanism. A transition to the DKP formalism, as well as a reconsideration of the role of the scalar field in quantum theory, could change the situation in this regard, and this will be the subject of further research.

Thus, perturbation theory in SM gauge field theory and in quantum gravity with gravitational regularization at Λ_g is finite and the theories are consistent, since the calculated values of the observables are not only finite, but also sufficiently small.

7. Discussion and conclusion

TSRT, formulated in the first article [8], is based on the general Lorentz group $O(1,3)$ including the time axis inversion and where antiparticles are described as particles of negative energy going, with their rest frames, backwards in time t.

The construction of RQM on the basis of TSRT leads to TS RQM, in which the main problems and contradictions of the former RQM disappear, and new TS RQM becomes a consistent and self-contained physical theory in the area of applicability of RQM.

The main results of TS RQM described in the article are the following:

1. The probabilities are positive, and the probability current of negative energy particles is negative due to the direction of the current against the time axis t.
2. This corrects the mistake of the former RQM boson theory, where the negative sign of the probability current was assigned to probability, and in TS RQM the boson theory became completely consistent for the first time;
3. This also corrected the inverse mistake in Dirac's theory, in which the sign of the probability current at negative energy is positive due to the normalization factor, although the eigenvalues of the current are negative, as required by TSRT, and in TS RQM the theory of fermions became completely consistent also for the first time;
4. In TS RQM, translational symmetry and the Hamilton-Jacobi formalism lead to a general wave equation for the wave function components of all free particles, in which the relativistic Hamiltonian appears quadratically, and KG equation, the differential form of this equation, expressing the relationship between energy and momentum, also becomes universal;
5. The average of 4-momentum and the probability current following from the KG equation also become general and describe motion of all free particles;
6. Linearization of the relativistic Hamiltonian leads to special wave equations with first derivatives (the DKP equations for spin 0, 1 and the Dirac equation for spin ½), which include mixing of the wave function components and describe spin and interactions;
7. From these special equations follow special probability currents, the drift parts of which, describing free particles, coincide with the general probability current, but there are also spin parts and parts with potentials;
8. The ZSF boundary conditions lead to causal propagators of particles;
9. The propagator approach with time-symmetric transition amplitudes and causal propagators leads to the standard diagram technique;
10. TS RQM takes into account the required by general relativity external gravitational field created by the total energy of the system of particles in any loop diagram;
11. This external field leads to a gravitational slowing down of proper times with freezing at the Planck length, the gravitational radius of the particle system;

12. This gravitational freezing leads to gravitational regularization of loop diagrams, making perturbation theory series in the propagator approach finite;
13. The effective coupling constants of quantum gravity and SM gauge fields are small at pre-Planck energies and therefore these theories are renormalizable;
14. The power-law growth of the loop contributions of the scalar field up to Planck scales indicates that this field is effective and their particle is composite.

All this demonstrates the fact that TS RQM, including the propagator approach, with its clear and practical formalism, free from contradictions, becomes as consistent science as QFT, giving a simpler and physically clearer description of processes with particles.

A more detailed presentation of TS RQM will be given in the book [17]. The third article [15] describes the basics of the time-symmetric QFT based on TSRT.

8. Appendix

8.1. Combining spinors into a tetraspinor

Since 4-inversion transforms u_p^r into v_p^r and vice versa, it is not sufficient to consider these spinors separately in TS RQM. Reflecting time axis leads to their mixing, and therefore we combine them into a single *tetraspinor* [7]:

$$w_p^r = \begin{pmatrix} u_p^r \\ v_p^r \end{pmatrix}. \tag{104}$$

Then the four Dirac equations for two types of spinors with $r = 1,2$:

$$(\gamma^\mu p_\mu - \kappa)u_p^r = 0, \qquad (-\gamma^\mu p_\mu - \kappa)v_p^r = 0 \tag{105}$$

take the form of two equations for two types of tetraspinors:

$$(\tau_3 \Gamma^\mu p_\mu - \kappa)w_p^r = 0, \tag{106}$$

where the matrices Γ^μ have the form:

$$\Gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & -\gamma^\mu \end{pmatrix} = \gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{107}$$

Solutions for two energy signs $w_{p\pm}^r$:

$$w_{p+}^r = \begin{pmatrix} u_p^r \\ 0 \end{pmatrix}, \qquad w_{p-}^r = \begin{pmatrix} 0 \\ v_p^r \end{pmatrix} \tag{108}$$

transform into each other under 4-inversion under the action of 4-inversion matrix Γ^5:

$$\Gamma^5 \begin{pmatrix} u_p^r \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ v_p^r \end{pmatrix}, \qquad \Gamma^5 \begin{pmatrix} 0 \\ v_p^r \end{pmatrix} = \begin{pmatrix} u_p^r \\ 0 \end{pmatrix}, \qquad \Gamma^5 = \begin{pmatrix} 0 & \gamma^5 \\ \gamma^5 & 0 \end{pmatrix} = \gamma^5 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{109}$$

Tetraspinors were introduced earlier for internal degrees of freedom, in particular isospin.

8.2. External gravitational field of particles in a loop diagram and gravitational self-regularization at the Planck scale

In a system of two or more interacting particles, particularly in the loop diagrams of propagator approach, the total energy of the system generates at each instant t an external gravitational field around the center of inertia of the system. This field slows down the local

proper times τ of quanta relative to t, leading to a gravitational redshift of these frequencies $\nu = \nu_0 g_{00}^{1/2}$ [14]. In this Appendix, we return to natural units, explicitly showing c and $\hbar$.

Gravitational redshifting, like the redshift in the clock paradox, is not an apparent effect, and on the hypersurface $t = const$ particles actually have lower frequencies $\nu < \nu_0$ than ν_0 of a distant particle. As the particle localization region decreases, ν reaches a maximum value $\nu_{\max} < \nu_g = \Lambda_g / h$ near l_g, and then rapidly decreases due to the strong gravitational redshift (ν_g is the Planck frequency). In a two-particle system, the redshift is small at interparticle distances $\sim 3l_g$, and at larger distances, $r \geq 3l_g$ the influence of gravity is weak, and the standard RQM is applicable.

The contribution to the amplitudes of processes in the interval $l_g \leq r \leq 3l_g$ is small and gives a small correction to the main contribution from the region $3l_g \leq r \leq \infty$. The gravitational radius of a two-particle system arising in the intermediate state of a one-loop diagram is of the order of $r_g \sim 2G \cdot 2h\nu / c^4$. For the estimation, it is sufficient to use the Schwarzschild metric, which yields for the frequency redshift ($\bar{\nu}_g \equiv \nu_g / 2$):

$$\nu = \nu_0 \left(1 - \frac{r_g}{r}\right)^{1/2} \simeq \nu_0 \left(1 - \frac{2G}{c^2}\frac{2h\nu}{c^2}\frac{\nu}{c}\right)^{1/2} = \nu_0 \left(1 - \frac{\nu^2}{\bar{\nu}_g^2}\right)^{1/2} \tag{110}$$

and the local frequency ν_0 and the frequency ν in terms t become related as follows:

$$\nu_0 \simeq \frac{\nu}{(1 - \nu^2 / \bar{\nu}_g^2)^{1/2}}, \quad \nu \simeq \frac{\nu_0}{(1 + \nu_0^2 / \bar{\nu}_g^2)^{1/2}}. \tag{111}$$

Even for $\nu_0 \gg \bar{\nu}_g$ the observed frequencies of each particle in the one-loop diagram $\nu < \bar{\nu}_g$, i.e., the total energy of a two-particle system tends to the Planck energy, which leads to the freezing of quantum fluctuations due to the closing to l_g the interparticle distances. For this reason, in the loop diagrams, integral over ν_0 in the interval $(0, \infty)$ is replaced, using (111), by integral over a finite interval $(0, \nu_g)$ with an upper limit ν_g. The differentials, taking into account (111), take the form:

$$d(\nu_0^2) \simeq \frac{d(\nu^2)}{(1 - \nu^2 / \bar{\nu}_g^2)^2}, \quad d(\nu^2) \simeq \frac{d(\nu_0^2)}{(1 + \nu_0^2 / \bar{\nu}_g^2)^2}. \tag{112}$$

In the second term of (112), the integrals over ν_0 explicitly converge due to the factor ν_0^{-4} and for $\nu_0 \gg \bar{\nu}_g$ all ν in the integrals tend to a constant value $\nu \to \bar{\nu}_g$.

In fact, gravitational regularization was previously used implicitly as the Pauli-Villars regularization, when the particle propagators were replaced by:

$$\frac{1}{p^2 - m^2c^2} \to \frac{1}{p^2 - m^2c^2} - \frac{1}{p^2 - M^2c^2} = \frac{1}{p^2 - m^2c^2} \cdot \frac{1}{1 - p^2 / M^2c^2}. \tag{113}$$

In one-loop diagrams, replacing both propagators as in (113) introduces into the integral a factor p^{-4}, which is equivalent to replacing (112).

Thus, at gravitational freezing of all processes near l_g and the energy Λ_g the events in the region $l \le l_g$ or $E_p > \Lambda_g$ do not occur during the finite interval of the world time of distant observers $t < \infty$ and do not contribute to the S -matrix.

We will write the general relativity restrictions in the form of *rules gravitational self-regularization*:

1. *Only processes in systems of particles with energies* $E_p < \Lambda_g$ *contribute to the transition amplitudes at distances between particles* $l > l_g$.
2. *The cutoff at* l_g *or* Λ_g *of gravity -free loop corrections provides upper limits for gravity-influenced corrections* $\Delta m_{\max}$, $\Delta e^2_{\max}$.
3. *The physical mass* m *and charge* e^2 *are the sums of the lower limits for the bare values* m_{0min} , e^2_{0min} *and upper limits for loop corrections* $\Delta m_{\max}$, $\Delta e^2_{\max}$:

$$\begin{aligned} m &= m_0 + \Delta m = (m_0 - \delta m) + (\Delta m + \delta m) = m_{0min} + \Delta m_{\max}, \\ e^2 &= e_0^2 + \Delta e^2 = (e_0 - \delta e^2) + (\Delta e^2 + \delta e^2) = e^2_{0min} + \Delta e^2_{\max}. \end{aligned} \tag{114}$$

Improvements in calculations will lead to a redistribution of small additions to the bare values and loop corrections. If the contribution of the upper limits to the total loop correction is small and sufficient for convergence of the perturbation theory series, then further refinements will only lead to a decrease in the absolute values of the corrections, leading to even faster convergence of the perturbation theory series.

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